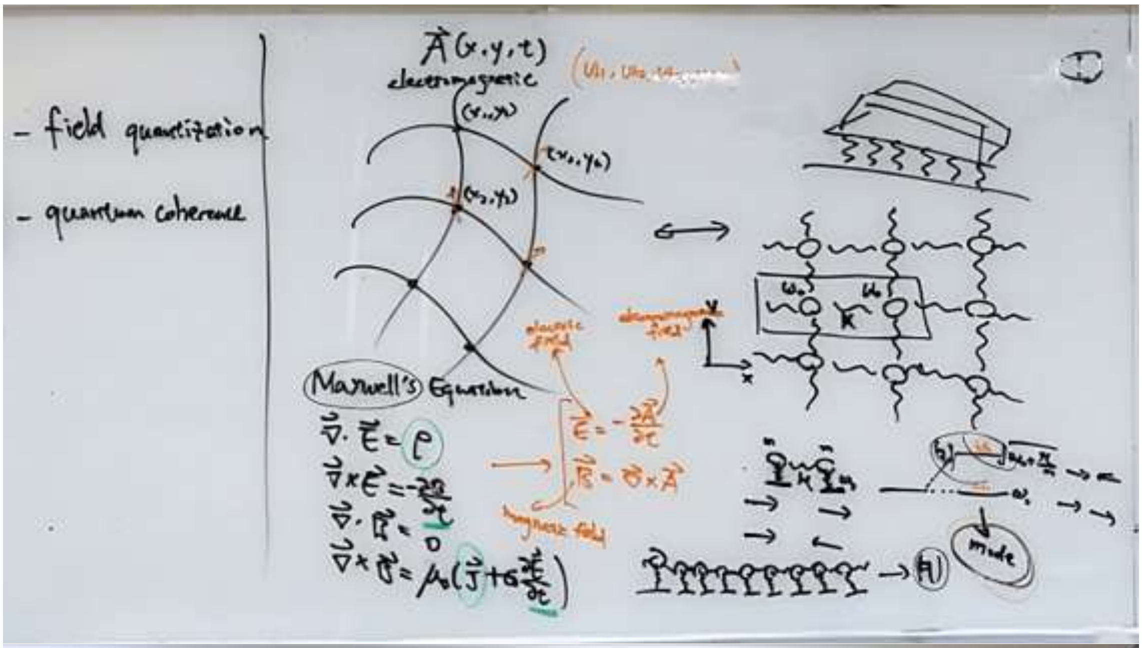
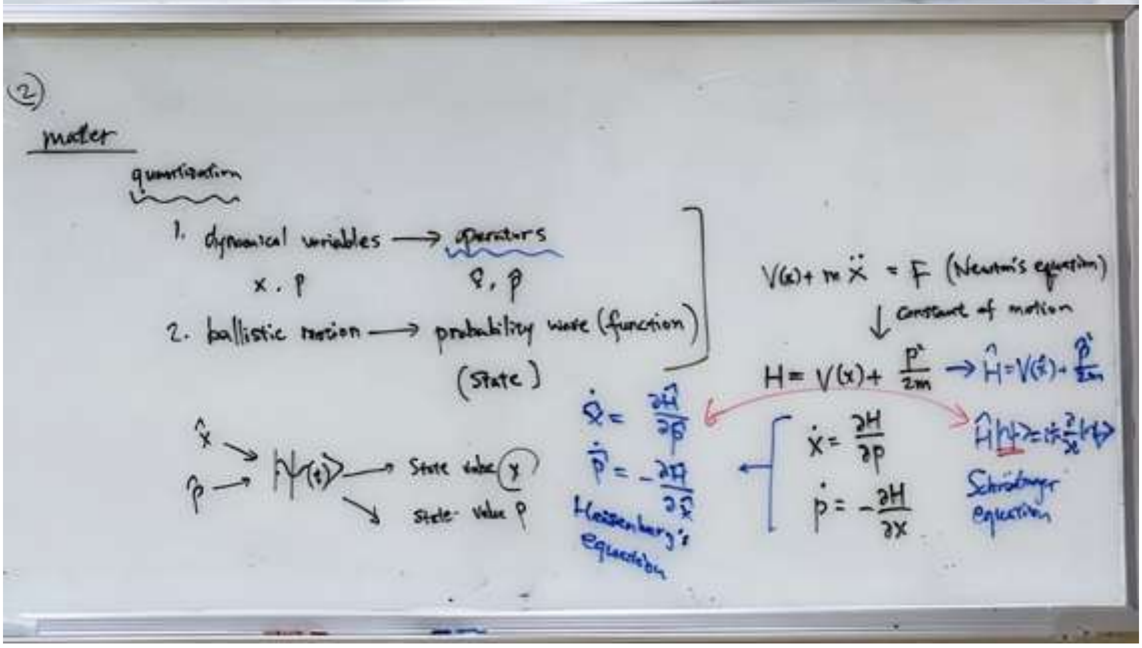
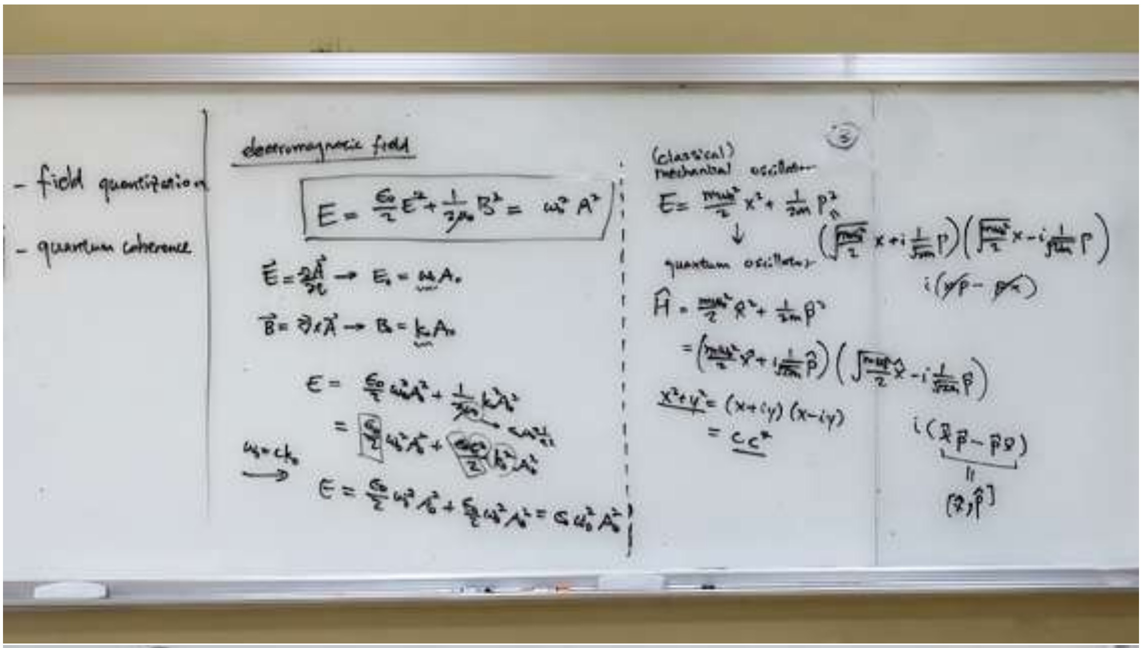
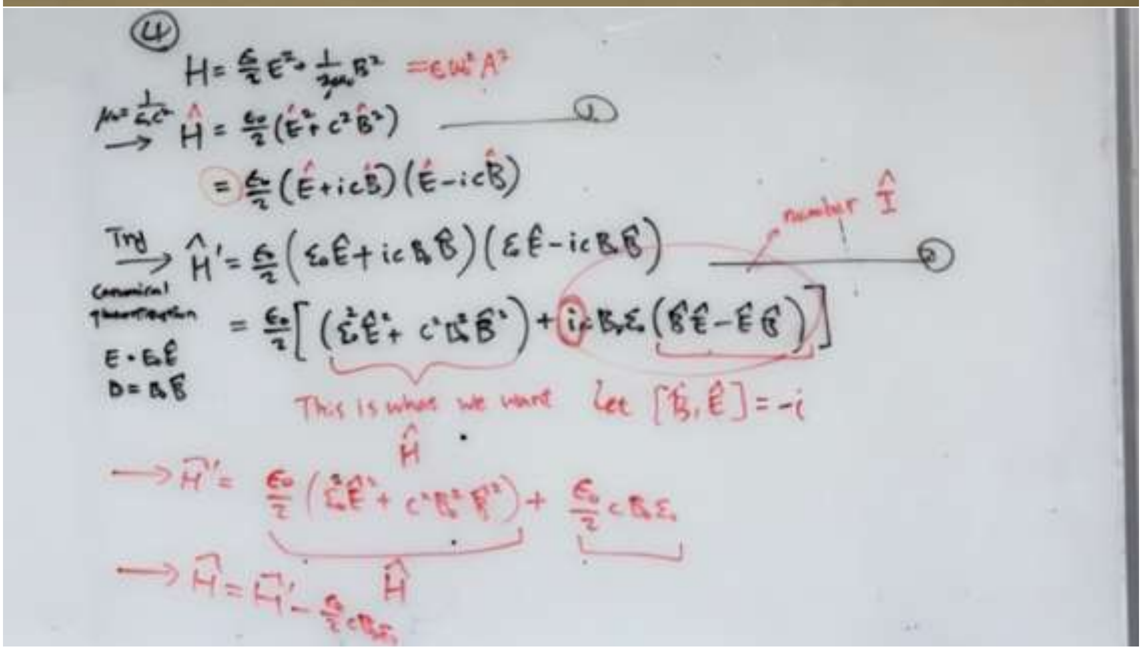
Lecture7 (optics) field quantization and quantum coherence






- field quantization
- quantum coherence
electromagnetic field
(classical) mechanical oscillator
quantum oscillator


④
canonical quantization
number
This is what we want
Let

- field quantization
- quantum coherence

⑤

$$① \rightarrow \hat{H} = \frac{\varepsilon_0}{2}\underbrace{\left(\mathcal{E}_0\hat{E} + ic\mathcal{B}_0\hat{B}\right)}_{\hat{A}}\underbrace{\left(\mathcal{E}_0\hat{E} - ic\mathcal{B}_0\hat{B}\right)}_{\hat{A}^+} - \frac{\varepsilon_0}{2}c\mathcal{B}_0\mathcal{E}_0$$

$\varepsilon_0, \mathcal{E}_0, \mathcal{B}_0, c$

$\xrightarrow{\text{Plane wave assumption}}$ $\hat{H} = \frac{\varepsilon_0\mathcal{E}_0^2}{2}\underbrace{(\hat{E} + i\hat{B})}_{\hat{a}}\underbrace{(\hat{E} - i\hat{B})}_{\hat{a}^+} - \frac{\varepsilon_0\mathcal{E}_0^2}{2}$

$\omega = ck$

$\mathcal{B}_0 = \frac{\mathcal{E}}{c}$

$$= \frac{\varepsilon_0\mathcal{E}_0^2}{2}(\hat{a}\hat{a}^+ - 1) \neq \frac{\varepsilon_0\omega_0^2A_0^2}{2}(\hat{a}\hat{a}^+ - 1)$$

$$\boxed{\hat{A} - \hat{a} = \hat{E} + i\hat{B}}$$

$\left[\begin{array}{l}\mathcal{E}_0 = \omega_0 A_0 \\ A \rightarrow A_0\hat{a}\end{array}\right.$

$$\rightarrow \hat{H} = \frac{\varepsilon_0\omega_0^2A_0^2}{2}\hat{a}\hat{a}^+ - \frac{\varepsilon_0\omega_0^2A_0^2}{2}$$

$$= \frac{\varepsilon_0\omega_0^2}{2}\underbrace{(A_0\hat{a})}_{\hat{A}}(A_0\hat{a}^+) - \frac{\varepsilon_0\omega_0^2A_0^2}{2}$$

⑥

electromagnetic field operator $A \rightarrow A_0\hat{a}$

electric field operator $E \rightarrow \mathcal{E}_0\hat{E}$

magnetic field operator $B \rightarrow \mathcal{B}_0\hat{B}$

$$\begin{cases}\hat{a} = \frac{1}{\sqrt{2}}\hat{E} + i\frac{1}{\sqrt{2}}\hat{B} \\ \hat{a}^+ = \frac{1}{\sqrt{2}}\hat{E} - i\frac{1}{\sqrt{2}}\hat{B}\end{cases}$$

$\hat{H}|\psi\rangle = E_n|\psi\rangle$

$E_n = \hbar\omega_0(n + \frac{1}{2})$

$n = 0, 1, 2, 3, 4 \ldots$

Using $\boxed{[\hat{B}, \hat{E}] = -i}$

$$[\hat{a}, \hat{a}^+] = \frac{1}{2}[\hat{E} + i\hat{B}, \hat{E} - i\hat{B}]$$

$$[\hat{E}, \hat{E}] = 0, \quad [\hat{B}, \hat{B}] = 0$$

$$\rightarrow [\hat{a}, \hat{a}^+] = \frac{1}{2}[\hat{E}, -i\hat{B}] + \frac{1}{2}[i\hat{B}, \hat{E}]$$

$$= \frac{-i}{2}[\hat{E}, \hat{B}] + \frac{i}{2}[\hat{B}, \hat{E}]$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

Therefore,

(Vacuum) zero-point energy fluctuation

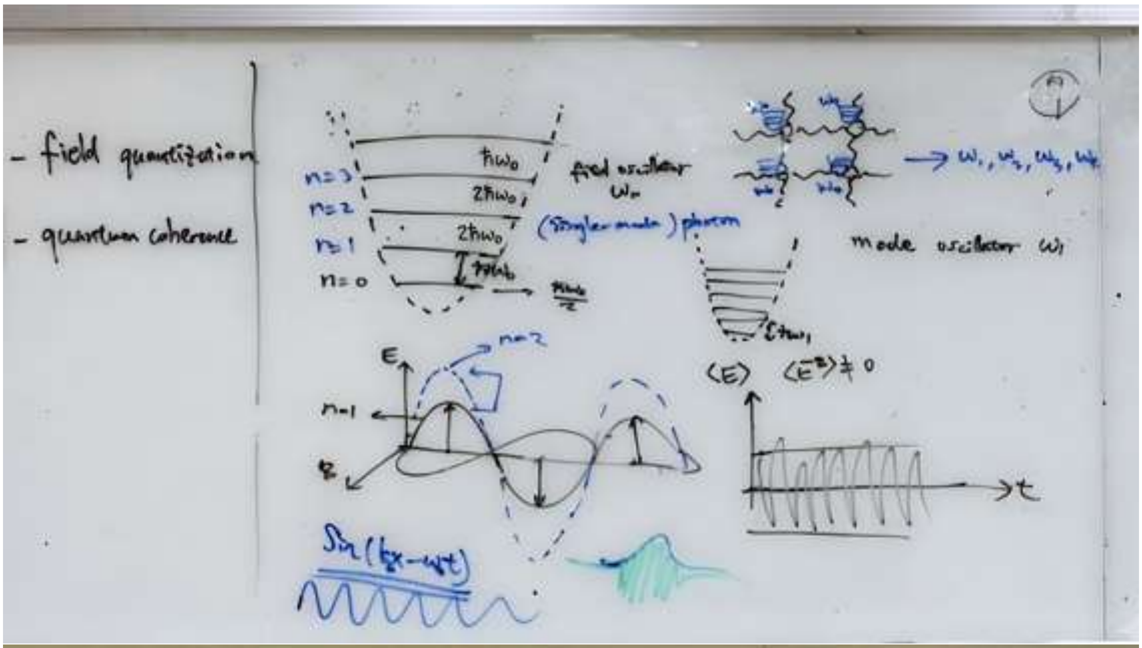
$$\hat{H} = \varepsilon_0\omega_0^2A_0^2(\hat{a}\hat{a}^+ - \frac{1}{2})$$

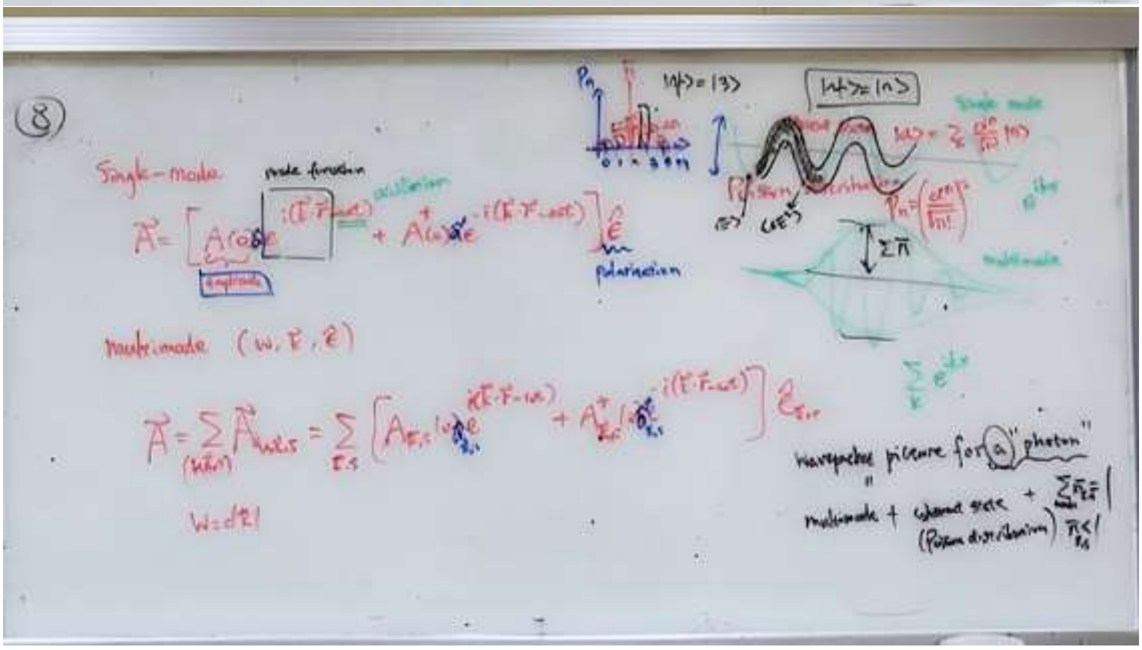
$$= \varepsilon_0\omega_0^2A_0^2(1 + \hat{a}^+\hat{a} - \frac{1}{2})$$

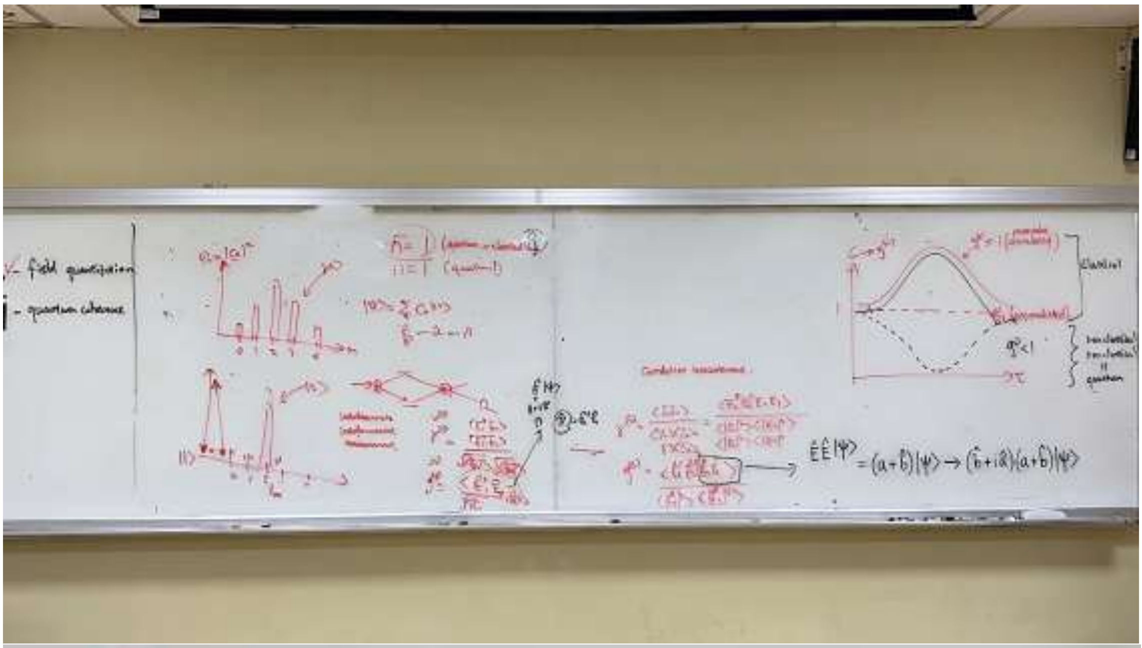
$$= \varepsilon_0\omega_0^2A_0^2(\underbrace{\hat{a}^+\hat{a}}_{\hat{n}} + \frac{1}{2}) = \boxed{\varepsilon_0\omega_0^2A_0^2\hat{a}^+\hat{a}} + \frac{\varepsilon_0\omega_0^2A_0^2}{2}$$

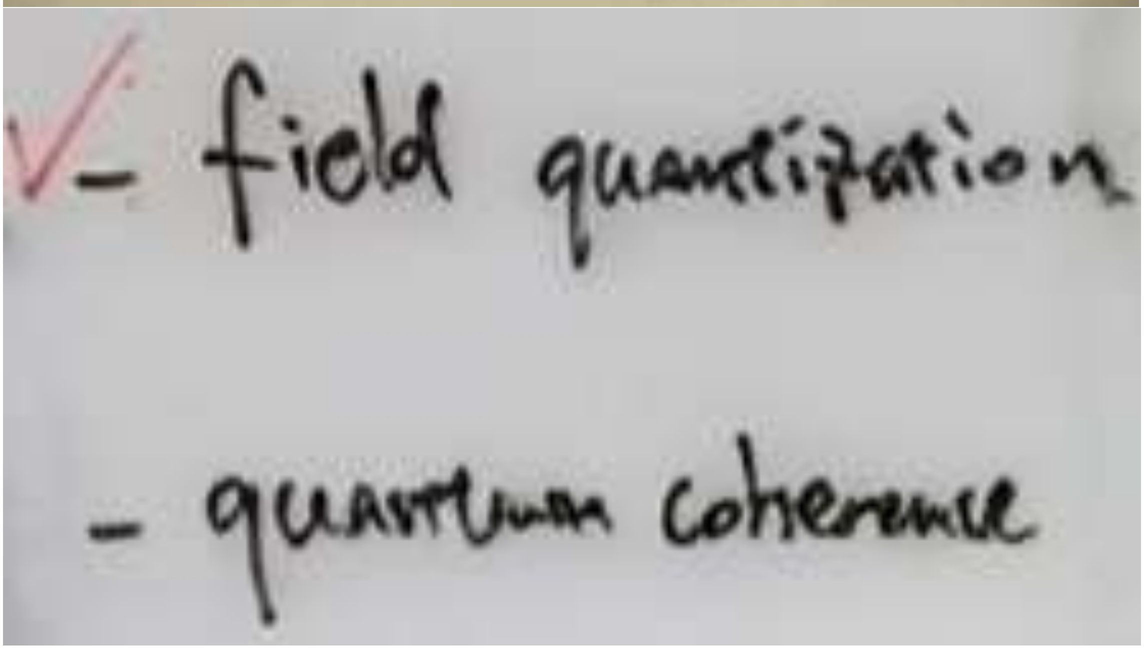
$\hat{n} = \hat{a}^+\hat{a}$

$$\rightarrow E_n = \hbar\omega_0(n + \frac{1}{2})$$


- field quantization
- quantum coherence
n=3
n=2
n=1
n=0
field oscillator
(single-mode) photon
mode oscillator ω1
⟨E⟩ ⟨E²⟩ ≠ 0
n=1
n=2
t
Sin(kx-wt)


Single-mode
mode function
oscillation
amplitude
polarization
multimode (ω, k, ε)
W=c|k|
|ψ⟩=|3⟩
|ψ⟩=|n⟩
single mode
multimode
wavepacket picture for a "photon"
multimode + coherent state
(Poisson distribution)

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